

APL105: Notes for fluid mechanics

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This document is a work in progress and will be updated as the chapter's lectures progress. Rough plan:

Topic	No. of Lectures
Classification of flows and properties of fluids	2
Kinematics: flow visualization, vorticity, irrotational flow.	3
Idealized flows : Euler equation and Bernoulli equation	2
Viscous flow: the Navier Stokes equation, solutions for flow through pipes,	4
Dimensional analysis, similarity and modelling of fluid flows	1
Total lectures	14

1 Introduction

There are three states of matter as we all know: solids, liquids and gases. There is, of course, further states of matter such as the plasma. But we can let that rest for the moment, and look at those states of matter that are immediately accessible to us. Why we know the differences between these states of matter, from our physical observation, it may also be interesting to know why they behave the way they behave. So, let us look at their qualitative properties from a molecular point of view. Solids typically have molecules organised

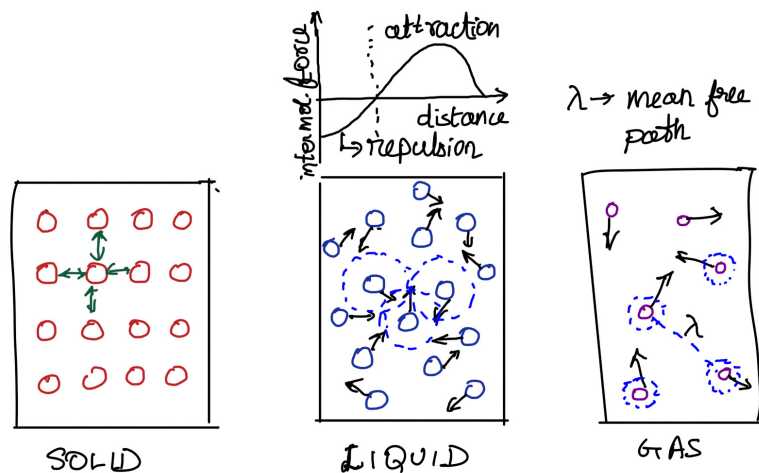


Figure 1: Three state of matter: molecular picture

in a lattice or in an amorphous but fixed arrangement. As we apply a force (or a stress) to the material, it deforms but the molecules are close together, their inter-molecular forces will try to pull and bring them back to their original configuration. Depending on the atomic (or molecular) properties and depending

on the temperature at which we find the material, the molecules may be farther apart than in the case of its solid state. They may be farther apart that they can't immediately pull each other and 'bring back' the original form. Such a loose state of being is called a fluid. When the molecules are far apart, but still feel the inter-molecular forces considerably, we call it a *liquid* state, and when the molecules are even farther apart that they don't feel the molecular forces, except when they come occasionally close to each other, we call the material to be in a *gaseous* state.

Our goal here is to study the mechanics of fluids, i. e., liquids and gases together. Mechanics involves both *statics* and *dynamics*. The former involves the study of stresses and forces acting on stationary systems whereas the latter involves the study of motion (kinematics) and the associated forces. Why do we study fluid mechanics?

Fluids, thanks to their ability to flow, can be quite useful for an engineer. Fluids are used for lubrication, to transform energy from one state to another, as medium in transport of chemicals and heat etc. In all of these applications fluids exert forces on objects (typically solid objects). Most of our interest, at least in this course, is to estimate the loads due to fluids on objects.

Since we established the scope of our study, it may be worthwhile to classify the different kinds of forces exerted by fluids. This way we might be able to devise different approaches to perform our analyses. Forces exerted on submerged bodies due to fluids is called *buoyancy* forces; force exerted on a submerged object but specifically owing to the flow of the fluid, but in a perpendicular direction is called *lift* force; force acting on submerged bodies due to the flow of the fluid but along the direction of the flow is called the *drag* force; force acting on any point on an object owing to the fluid around it is called the *pressure* force, the force acting on a surface tangential to the flow direction, akin to friction in solids, is called a shear force. The effect of these forces may be to displace the solid, deform it or to cause failure.

1.1 Definition of a fluid

The mechanistic definition of a fluid is the following:

A fluid is a substance that *continuously* deforms under the application of shear force, *however small the force may be*.

Note the italicised parts of the definition. It is not sufficient that the material deforms, but it needs to deform continuously. Let us take the example of spill of oil on the floor and the instance when you step foot on this layer of liquid. Since you were walking, you had a horizontal velocity and that means you have a horizontal momentum. The top surface of the liquid that comes in contact with your shoe experiences a forces tangential to this surface, hence it is a shear force. If there was a layer of rubber (a solid) instead of the oil spill, the shear force would have cause the rubber sheet to deform, but the deformation comes to stop and eventually arrest the horizontal slip of your foot. This means the deformation would have been non-continuous, unless the forces was too much that the rubber tore or slipped off of the floor. But oil, being a liquid would deform continuously. And you would have slipped continuously however small the force might have been. If the shear force was low, the slip might have been slow. So the effect of shear is to impart a velocity, and not just a deformation. Velocity is deformation per unit time, and so we would call it *rate of deformation*. Thus the fluid exhibits its resistance to shear force through a rate of deformation and not through deformation alone. Compare it with solid where it resists a force with deformation.

1.2 Newton's law of viscosity

Shear stress cause a strain rate to occur in a fluid. Let us *model* the slipping scenario. Somewhere within a liquid that is getting sheared, let us take a small rectangular element of height δy . Relative to its bottom lets assume the top surface of this element *continuously* deforms. At a time interval δt , lets say the velocity of the top surface changes by δu . The shear strain experienced by this fluid element, assuming small deformations is:

$$\theta \approx \tan \theta = \frac{\delta u \delta t}{\delta y}.$$

As this time interval diminishes to 0, $\lim_{\delta t \rightarrow 0}$, we can calculate the shear strain rate as:

$$\dot{\theta} = \lim_{\delta t \rightarrow 0} \frac{\theta}{\delta t} = \frac{du}{dy}.$$

Newton's law states that:

$$\tau \propto \frac{du}{dy} = \mu \frac{du}{dy}.$$

The constant of proportionality μ is called the coefficient of viscosity, and has the S. I. unit of Pa·s.

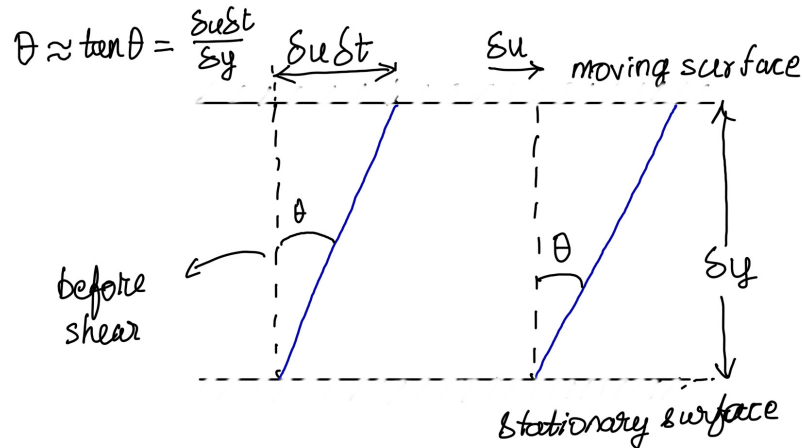


Figure 2: Shear force and deformation rate

Here the stress is written as a *scalar* quantity. However one can easily realize that a force can act either normally or tangentially to a plane and a plane can be oriented with its normal along any of the coordinate direction. We will look at these possibilities in a short while. But at the moment it is sufficient to notice that a stress element can only be defined corresponding to a surface and a force's component. So τ is denoted with two subscripts as τ_{ij} , where the first index i tells us the direction of the normal to the surface, and the second index j tells us the direction at which the force is acting on this surface. This will be explained in detail when we derive stress *tensor*.

Figure 3:

$$\sigma_{ij} = \frac{u}{j}.$$

2 Lecture 2

In the previous lecture

2.1 Lecture 2

- revisit Newton's law of viscosity

- linear profile of velocity
- examples of non-linear profile - plane Poiseuille flow
- flow can be caused by pressure difference
- stress - strain rate relationship : Ideal, Newtonian, Dilatant or shear thickening (wet sand), Ideal, pseudo plastic or shear thinning, Bingham fluid.
- Continuum in a fluid: properties $\mu, \rho, T, \vec{V}, p$. Air, linear size is one micron, we have 3×10^7 molecules.
- $\lambda/a \ll 1$ Knudsen number. continuum valid. - rarefied gas dynamics
- molecular dynamics
- some thermodynamics
- Kinematic descriptions : Particle approach - may not be the best approach
- Field approach where a fluid occupying (x, y, z) at a time t has property $\beta(x, y, z)$. Put many probes in different points in the flow.
- If β depends only on space and not on time, the flow is steady.
- Flow in a converging channel. V does not change in a location. Flow is steady. But velocity changes with space.
- 1D, 2D, 3D flow.
- visualization: Smoke from chimney, incense stick. Streamlines, streaklines and pathlines.
- streamline

$$\frac{dy}{dx_{streamline}} = \frac{v(x, y, t)}{u(x, y, t)}.$$

- pathline: path followed by a fluid particle. Specify the particle.

$$\frac{dx_p}{dt} = u(x_p, y_p, t) \frac{dy_p}{dt} = v(x_p, y_p, t)..$$

At $t = t_0$ particle is at y_0

$$x = f(t)y = g(t).$$

- Streakline. If particles have passed through a point x_0, y_0 , at a later time t , join all these particles and this line is called a streakline. Particles pass through (x_0, y_0) at time τ . $t_0 \leq \tau \leq t_F$. Easiest to visualize. Put a dye injector in a flow. Put a wire and pass current, hydrogen bubbles are released.
- In a steady flow, streamlines, streaklines and pathlines are identical.
- Lagrangian and Eulerian description. Label $X(t_0) = X_0$. $X(X_0, t)$ gives X
- Eulerian description. Field approximation.
- In steady flow all lines are the same.
- Steady flow is not a function of time.

2.2 Lecture 3

- Gauss divergence theorem. Closed volume V and surface area A .
- Infinitesimal volume dV
- Sum over all such volume elements $\rightarrow$ volume integral.

$$\iiint_V \Phi dV.$$

- Surface area integration: Partial area

$$\iint_A \Phi dA.$$

Full area:

$$\oint_A \Phi dA.$$

where Φ is a field.

- Divergence theorem

$$\iiint_V \nabla \cdot \vec{B} dV = \oint_S (\vec{B} \cdot \hat{n}) dA.$$

- Generalized version: it can be applied not only to a vector, but to any order tensor (scalar, tensor etc).
- Divergence can be replaced by gradient.

$$\iiint_V \nabla \Phi dV = \oint_S \Phi \hat{n} dA.$$

- check if quantities are uniformly vector or scalar on both sides.
- Hydrostatics: Static fluid. Fluid where there is no motion. Fluid as a whole may move, but there is no flow. Pressure: normal force per unit area. Pressure is also a stress (τ_{nn}). Sometimes we use the symbol σ_{nn} . Which is also used in solid mechanics. By definition n points outwards, and pressure is defined as normal force pointing inwards per unit area. so

$$\sigma_{nn} = -p.$$

- Under the action of a normal force, it is possible that a fluid doesn't deform. Refer to definition of what is a fluid.
- topic: stress tensor
- stress is force per area.
- Force has a direction, area has a direction too. Draw volume element and normal at dA . $dA \hat{n}$ is a vector.
- 3 components of F and 3 components of dA
- total of nine combinations for a stress.

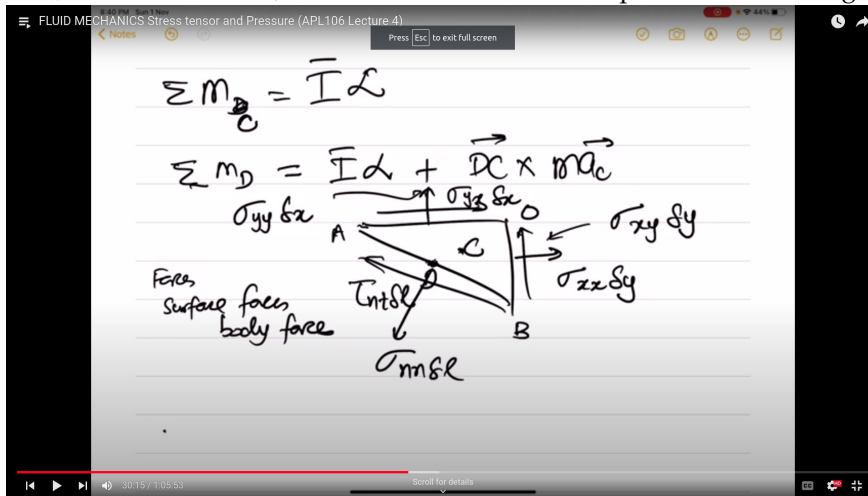
$$\begin{bmatrix} F_x/dA_x & F_x/dA_y & F_x/dA_z \\ F_y/dA_x & F_y/dA_y & F_y/dA_z \\ F_z/dA_x & F_z/dA_y & F_z/dA_z \end{bmatrix}.$$

- second order tensor
- formed by 2 different vectors.
- σ_{ij} : i is direction of area and j is direction of force.
- Stress at a point.
- Triangular area $\delta x, \delta y, \delta l$. Normal to the area points to positive direction, it is a positive face. So positive face and positive force gives positive stress term. If positive and negative, then stress is negative.
- For the triangle are AOB, where A is top. θ is between AO and AB (hypotenuse and length dl). on OB: upward force is $\sigma_{xy} \cdot \delta y \cdot 1$ rightward force is $\sigma_{xx} \cdot \delta y \cdot 1$. For the dl face: $\sigma_{nn} \cdot dl \cdot 1$ is the normal and for tangential (draw angular upwards) $\sigma_{nt} \cdot dl \cdot 1$
- force balance is $\sigma F = m \mathbf{a}$. Any other forces?
- weight of the element. $W = volume * \rho * g$ acting at centroid C.
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$$\sigma_{xx} \delta y + \sigma_{yx} \delta x - \sigma_{nt} \delta l \cos \theta - \sigma_{nn} \delta l \cos \theta - \rho g \delta x \delta y / 2 = \rho \delta x \delta y a_y / 2.$$

- $\cos \theta = \delta x / \delta l$ and $\sin \theta = \delta y / \delta l$...
- Let $\delta x \rightarrow 0$ and $\delta y \rightarrow 0$ then terms containing the stress terms contain single δ length term. But accelerations are multiplied by area terms. In the limit of shrinking the volume to a point, only the σ terms remain. The other terms become very small.
- $\sigma_{yx} \cos \theta + \sigma_{xx} \sin \theta = \sigma_{nt} \cos \theta + \sigma_{nn} \sin \theta$ for x direction and
- $\sigma_{yy} \cos \theta + \sigma_{xy} \sin \theta = -\sigma_{nt} \sin \theta + \sigma_{nn} \cos \theta$. then we can find σ_{nt} and σ_{nn} for any surface. For 3D too, this is applicable.
- $\Sigma M_c = I \alpha$

- $\Sigma M_D = I\alpha + DC \times ma_c$ where c is centroid, d is midpoint of the slanting face.



- moment due to shear force on OB is clockwise and $\sigma_{xy}\delta y\delta x/2$ and moment due to shear force on OA is $\sigma_{yx}\delta x\delta y/2$.
- moments due to gravity, $I\alpha$ and $ma \times CD$
- σ_{xy}, σ_{yx} contains 2nd order
- gravity contains 3rd order.
- $ma \times cd$ will contain 3rd order term
- I is $m l^2$ and acceleration /length will have 4 delta terms.
- when delta tends to 0 for all lengths, we get, only σ_{xy}, σ_{yx} gives us a symmetry of stress.
- sum of moments = $I\alpha$ is what we used.
- If there are body couples in the fluid, this may not be of order 3 or 4 and may not be symmetric. Otherwise it is a fairly general result.
- if we rotate the axes we find the trace of the stress tensor is independent of rotation.
- $T/3\delta_{ij}$ property of kronecker brings the isotropy. We call this isotropic stress. The remaining (show) is deviatoric part. What is the trace of deviatoric part?
- show tensorially ... σ_{ij} is minus $p\delta_{ij}$

3 Lecture 4

- Hydrostatics. analysis of fluid without flow.
- total force on an infinitesimal volume due to pressure written as an integral over area.

$$F = \oint_S -p \hat{n} dS.$$

- Fluid is like a rigid body and is static. Lets say it is accelerating with $\vec{a}$
- deviatoric stress is 0. Then, for the volume V we have $F_S + F_B = m\vec{a}$. Newton's second law
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$$-\oint_S p \hat{n} dS + \iiint_V \rho \vec{g} dV = \iiint_V \rho \vec{a} dV.$$

- Divergence theorem

$$-\oint_S p \hat{n} dS = - \iiint_V \nabla p dV.$$

- Then we have

$$\iiint_V (-\nabla p dV + \rho g dV) = \iiint_V \rho \vec{a} dV.$$

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$$\iiint_V (-\nabla p + \rho g - \rho \vec{a}) dV = 0.$$

- Note the direction. The above hold for vertical direction. For horizontal direction g will be zero.
- Apply the localization theorem : $\boxed{-\nabla p + \rho g = \rho \vec{a}}$