MTL766: Multivariate statistics

Homework I

1. Applied Multivariate Statistical Analysis book by Johnson and Wichern, p. 200-205 : Exercises 4.1 to 4.9, 4.14 to 4.17.

2. Suppose $Y \sim N_3(\mu, \Sigma)$, where

$$\mu = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}; \Sigma = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 13 & 4 \\ -2 & 4 & 4 \end{pmatrix}.$$

   (a) Find the distribution of $z = 2Y_1 - Y_2 + 3Y_3$.

   (b) Find the joint distribution of $Z_1 = Y_1 + Y_2 + Y_3$ and $Z_2 = Y_1 - Y_2 + 2Y_3$.

   (c) Find the distribution of Y_2.

   (d) Find the joint distribution of Y_1 and Y_3.

   (e) Find the joint distribution of Y_1, Y_3, and $(Y_1 + Y_2)/2$.

3. Show that if $X \sim N_p(\mu, \Sigma)$ and given some matrices A and B, then AX and BX are independent if and only if $A\Sigma B = 0$.