Jan 21

(I) Random variable

(II) Cumulative distribution function of a RV

Theorem (without proof).
There is one-to-one correspondence between $\mathbb{P}_X$ and F_X.

Remark: The probability law of a RV X is known if we know either of $\mathbb{P}_X$ or F_X.

Types of RV

(i) Discrete RV
(ii) Continuous RV
(iii) Other.

Discrete RV:

Def (I): The set of all possible values of RV X is countable.

$\boxed{\text{eg}}$: (I) X = number of heads in 1000 tosses of a coin

possible value set
$\{0, 1, 2, \ldots, 1000\}$

(11) $\underline{\Omega = \mathbb{N} \cup \{0\}}$

$\mathcal{C} = 2^{\Omega}$, $\underline{\lambda > 0 \text{ fixed}}$

$$\mathbb{P}(\{x\}) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad \forall x \in \Omega$$

$$\mathbb{P}(A) = \sum_{x \in A \cap \Omega} \mathbb{P}(\{x\})$$

verify

$$\sum_{x \in \Omega} \mathbb{P}(\{x\}) = 1$$

valid prob. measure.

Define RV ,

$X : \Omega \to \mathbb{R}$

$X(\omega) = \omega$, identity map

$X(\omega) =$

You can check X is a random variable on $(\Omega, \mathcal{F}, \mathbb{P})$

Possible value set for X

$\underbrace{\{0, 1, 2, \dots\}}$

infinite set.

Characterising property of CDF (all kind of RVs)

Theorem (without proof)

A function $F : \mathbb{R} \to \mathbb{R}$ is a CDF corresponding to a Random variable if it satisfy following

(i) $\lim_{x \to -\infty} F(x) = 0$

(ii) $\lim_{x \to \infty} F(x) = 1$

(iii) $F(x)$ is a non-decreasing function

(iv) $F(x)$ is a right continuous function.

$$\boxed{F_X(x) = P(X \leq x)}$$

Ex: Tossing A two

fair coins

$\Omega = \ldots$

$\mathcal{F} = 2^{\Omega}$

$\mathbb{P}$ — equally likely

X = number of heads

$$F_X(x) = \begin{cases} 0 & , \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 1 \\ \frac{3}{4} & \text{if } 1 \le x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$$

All characterizing properties of CDF are satisfied.

Discrete RV

Let X be a discrete RV with probability law

$\mathbb{P}(X = x) = p_x$ where $x \in S$ — possible value set

where $p_x \geq 0$

$$\sum_{x \in S} p_x = 1$$

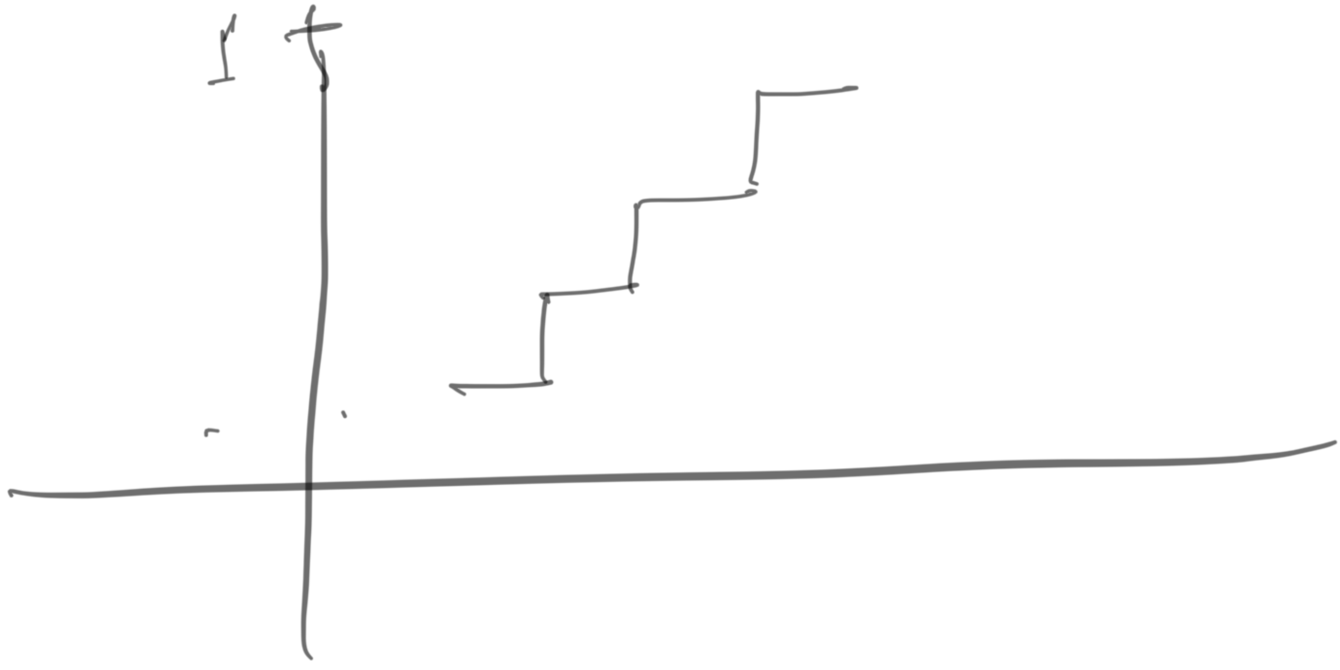
Eg $\mathbb{P}(X \in A)$

$$= \sum_{x \in A \cap S} \mathbb{P}(X = x)$$

Lemma: The CDF of X,

$$F_X(x) = \mathbb{P}(X \le x) = \sum_{y \le x} p_y$$

Lemma For a discrete RV X, the CDF F_X is a step function.



Probability mass function

(PMF)

PMF of a discrete RV X is probabilities associated with singleton value sets.

In the preceding construct. p_x is the PMF.

Eg: Possible value set (Support) of a RV X be $S = \mathbb{N} \cup \{0\}$ with probability law

$$\mathbb{P}(X = x) = p_x = \begin{cases} \dfrac{e^{-\lambda} \lambda^x}{x!}, & x \in S \end{cases}$$

$\begin{cases} \; \\ 0 \quad , \text{otherwise} \end{cases}$

Remark: To describe a discrete RV, we should know one of the following

(i) Probability measure

(ii) CDF

(iii) PMF

Lemma: For a discrete RV X, with CDF F_X and PMF P_X,

$$p_X(x) = \mathbb{P}(X = x)$$

$$= F_X(x) - \underbrace{\lim_{y \uparrow x} F_X(y)}_{\text{left hand limit at } x}$$

$$\mathbb{P}(X \leq x) - \mathbb{P}(X < x)$$

$$= \mathbb{P}(\{X = x\})$$

$$\mathbb{P}(A) = \mathbb{P}(A \setminus B) + \mathbb{P}(A \cap B)$$

Continuous RV:

Def: let X be a RV with CDF F_X.
X is said to be a continuous RV if F_X is a continuous function.

Eg: support set of RV X
set of possible values
is $[0,1]$ and

CDF
$$F_X(x) = \text{Length}\left(\underline{[0,1] \cap (-\infty, x]}\right)$$
$$= \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x < 1 \end{cases}$$

$1, \quad \text{if } x \geq 1$

Clearly F_X is a valid CDF and it is a continuous function.

So, X is a continuous RV.

Continuous RV

↓ Absolutely continuous

↓ Singular continuous → out of scope

Remark: In this course continuous RV means absolutely continuous RV.

Probability density function (PDF) next lecture