Jan 20

Monte Hall problem

Assumption:

(i) Car is equally likely behind any of the doors

(2) The contestent choses the door randomly initially.

(3) Others

Formalise

Wlog:

(i) The Cont. initially choses door 1.

(ii) The Car is behind any of

the doors with equal prob.

C_i = event that car is behind door i

$$\mathbb{P}(C_i) = \frac{1}{3}, \quad i = 1, 2, 3$$

(iii) The host (Monte) opens door 2. , this event we denote by H_2.

The probability we are interested in

$\mathbb{P}(C_1 | H_2)$ — prob. of win by staying

$\mathbb{P}(C_3 | H_2)$ - prob. of win by switching

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we have

we have

$$\mathbb{P}(C_i) = \frac{1}{3}, \quad i = 1, 2, 3$$

$$\mathbb{P}(H_2 \mid C_1) = \frac{1}{2}$$

$$\mathbb{P}(H_2 \mid C_2) = 0$$

$$\mathbb{P}(H_2 \mid C_3) = 1$$

$$\begin{aligned}\mathbb{P}(H_2) &= \mathbb{P}(C_1) \cdot \mathbb{P}(H_2 \mid C_1) \\ &\quad + \mathbb{P}(C_2) \cdot \mathbb{P}(H_2 \mid C_2) \\ &\quad + \mathbb{P}(C_3) \cdot \mathbb{P}(H_2 \mid C_3) \\ &= \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 0 \\ &\quad + \frac{1}{3} \cdot 1 \\ &= \frac{1}{6} + \frac{1}{3} = \frac{1}{2}\end{aligned}$$

$$\mathbb{P}(C_1 \mid H_2) = \frac{\mathbb{P}(C_1) \cdot \mathbb{P}(H_2 \mid C_1)}{\mathbb{P}(H_2)}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$$

$$\mathbb{P}(C_3 \mid H_2) = \frac{\mathbb{P}(C_3) \cdot \mathbb{P}(H_2 \mid C_3)}{\mathbb{P}(H_2)}$$

$$= \frac{\frac{1}{3} \cdot 1}{\frac{1}{2}}$$

$$= 2/3$$

3

Random Variable

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.

Def: A random variable X on the measurable space $(\Omega, \mathcal{F})$ is a function $X: \Omega \to \mathbb{R}$ such that for any $a \in \mathbb{R}$

$$X^{-1}((-\infty, a]) \in \mathcal{F}$$

where

$X^{-1}((-\infty, a]) = \{\omega \in \Omega : X(\omega) \le a\}$

Eg: Tossing of two fair coins

$\Omega = \{HH, HT, TH, TT\}$

$\mathcal{F} = 2^{\Omega}$

$\mathbb{P}$ — equally likely

$X : \Omega \to \mathbb{R}$

$X(\omega)$ = number of heads in ω

ω	HH	HT	TH	TT
$X(\omega)$	2	1	1	0

$X^{-1}(\{0\}) = \{TT\}$

$$X^{-1}(\{1\}) = \{HT, TH\}$$

$$X^{-1}((-\infty, 0]) = \{TT\}$$

Observe that

$$\underline{X^{-1}(\underline{(-\infty, a]})} = \begin{cases} \phi & \text{if } a < 0 \\ \{TT\} & \text{if } 0 \leq a < 1 \\ \{TT, HT, TH\} & \text{if } 1 \leq \underline{a < 2} \\ \Omega & \text{if } a \geq 2 \end{cases}$$

$$\in \Sigma$$

Induced probability measure of a induced random variable

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a prob. space and X be random variable defined on $(\Omega, \mathcal{F})$.

The induced prob. measure of random var. X by $(\Omega, \mathcal{F}, \mathbb{P})$ is denoted by $\mathbb{P}_X$ and defined as

$$\mathbb{P}_X(A) = \mathbb{P}(X^{-1}(A))$$

for any $A \in \mathbb{B}(\mathbb{R})$.

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preceding ex.

take $0 \le a < 1$,

$\mathbb{P}_X((-\infty, a])$

$= \mathbb{P}(X^{-1}(-\infty, a])$

$$= \mathbb{P}(\{TT\}) = \frac{1}{4}$$

Cumulative distribution function (CDF)

CDF of a random var. X is a function

$F_X : \mathbb{R} \to [0,1]$ defined as

$$F_X(a) = \mathbb{P}_X((-\infty, a])$$

$$= \mathbb{P}_X(X \leq a)$$

Theorem (without proof):

there is one to one

" correspondence between F_X and $\mathbb{P}_X$.

eg. Contd.

$$F_X(a) = \mathbb{P}_X(X \leq a)$$

$$= \begin{cases} 0 & \text{if } a < 0 \\ \frac{1}{4} & \text{if } 0 \leq a < 1 \\ \frac{3}{4} & \text{if } 1 \leq a < 2 \\ 1 & \text{if } a \geq 2 \end{cases}$$

Types of random var. (RV)

① Discrete R.V.

When possible values of RV make a countable set.

eg. previous example, number of infected person by a disease in Delhi

② Continuous R.V.

When possible values of RV make an uncountable set

Eg Height.

3 Other