

# MTL762: Probability Theory (3-0-0)

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## Course Content:

- Limits of sequences of sets,  $\sigma$ -field of events. Probability measure, Probability space.
- Random variables, Induced probability space, Probability distribution, Decomposition theorem. Expectations and moments.
- Independence of events and random variables, Borel-Cantelli lemmas, Tail  $\sigma$ -algebras, and Kolmogorov's zero-one law.
- Convergence theorems for expectations of sequences of random variables (monotone convergence theorem, Fatou's lemma, dominated convergence theorem).
- Various modes of convergence of sequences of random variables (almost surely, in probability, in  $r^{th}$  mean).
- Convergence of sequences of distribution functions, Helly-Bray theorems, convergence of moments, Scheffe's theorem.
- Characteristic function and its properties, inversion formulae.
- Some inequalities (Markov, Jensen and Holder), Khintchin's weak law of large numbers, Kolmogorov strong law of large numbers. Central limit theorems of Lindeberg-Levy, Liapounov and Lindeberg-Feller.
- Multivariate random variables, marginal and joint distributions, covariance and correlation matrices. Transformation of multivariate random variables.

## References:

1. K. B. Athreya and S. N. Lahiri, Measure theory and probability theory, New York: Springer, 2006.
2. K. L. Chung, A course in probability theory, Academic Press, 2001.

## Supplementary texts:

1. D. Stirzaker, Elementary probability, Cambridge University Press, 2003.
2. P. Billingsley, Probability and measure, John Wiley and Sons, 1995.
3. Robert B. Ash and A. Doléans-Dade Catherine, Probability and measure theory, Academic press, 2000.

**Evaluation:**

- Minor I: 25 marks.
- Assignments ( $2 \times 5 = 10$ ): There will be 2 assignments, 5 marks in each.
- Quizzes ( $2 \times 10 = 20$ ): 2 Quizzes, 10 marks in each.
- Presentation: 10 marks.
- Major: 35 marks.

**Attendance and other policies:**

- Cell phones and laptops are discouraged during class.
- While there is no weightage for attendance, students are highly encouraged to attend all lectures.
- There will be no make-up opportunity for the quizzes, assignments, or presentation.
- Discussion/collaborations for solving the assignments is encouraged. However, students are expected to write down the solutions on their own.
- Minor and Major exams will be open-book.