MTL108: Solution to Problem Set-11

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Problems

1. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = 0.1$ mg. Suppose that the results of five successive weighings of the same object are as follows:

$$3.142,\ 3.163,\ 3.155,\ 3.150,\ 3.141.$$

(a) Determine a 95% confidence interval estimate of the true weight.

(b) Determine a 99% confidence interval estimate of the true weight.

Solution: Given: $\sigma = 0.1$ mg, $n = 5$, measurements: $3.142,\ 3.163,\ 3.155,\ 3.150,\ 3.141$

$$\bar{x} = \frac{3.142 + 3.163 + 3.155 + 3.150 + 3.141}{5} = \frac{15.751}{5} = 3.1502$$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.1}{\sqrt{5}} = \frac{0.1}{2.2361} = 0.04472$$

Since σ is **known**, we use the **Z-distribution**.

(a) 95% Confidence Interval

Critical value: $z_{\alpha/2} = z_{0.025} = 1.96$

$$CI = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 3.1502 \pm 1.96 \times 0.04472 = 3.1502 \pm 0.0877$$

$$\boxed{(3.0625, \quad 3.2379)}$$

(b) 99% Confidence Interval

Critical value: $z_{\alpha/2} = z_{0.005} = 2.576$

$$CI = 3.1502 \pm 2.576 \times 0.04472 = 3.1502 \pm 0.1152$$

$$\boxed{(3.0350, \quad 3.2654)}$$

2. The PCB concentration of a fish caught in Lake Michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of 0.08 ppm (parts per million). Suppose the results of 10 independent measurements of this fish are

$$11.2,\ 12.4,\ 10.8,\ 11.6,\ 12.5,\ 10.1,\ 11.0,\ 12.2,\ 12.4,\ 10.6.$$

(a) Give a 95% confidence interval for the PCB level of this fish.

(b) Give a 95% lower confidence interval.

(c) Give a 95% upper confidence interval.

Solution: $\sigma = 0.08$ ppm, $n = 10$

Measurements: 11.2, 12.4, 10.8, 11.6, 12.5, 10.1, 11.0, 12.2, 12.4, 10.6

$$\bar{x} = \frac{11.2 + 12.4 + 10.8 + 11.6 + 12.5 + 10.1 + 11.0 + 12.2 + 12.4 + 10.6}{10} = \frac{114.8}{10} = 11.48$$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.08}{\sqrt{10}} = \frac{0.08}{3.1623} = 0.02530$$

(a) 95% Two-Sided Confidence Interval

Critical value: $z_{\alpha/2} = z_{0.025} = 1.96$

$$CI = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 11.48 \pm 1.96 \times 0.02530 = 11.48 \pm 0.0496$$

$$\boxed{(11.4304, \quad 11.5296)}$$

(b) 95% Lower Confidence Interval (One-Sided Lower Bound)

We are 95% confident the true mean is **at least** this value.

Critical value: $z_\alpha = z_{0.05} = 1.645$

$$\bar{x} - z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 11.48 - 1.645 \times 0.02530 = 11.48 - 0.04162 = 11.4384$$

$$\boxed{\mu \geq 11.4384}$$

(c) 95% Upper Confidence Interval (One-Sided Upper Bound)

We are 95% confident the true mean is **at most** this value.

$$\bar{x} + z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 11.48 + 1.645 \times 0.02530 = 11.48 + 0.04162 = 11.5216$$

$$\boxed{\mu \leq 11.5216}$$

3. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90% confidence interval estimate for the average score of all students.

Solution: $\sigma = 11.3$, $n = 81$, $\bar{x} = 74.6$, Confidence level = 90%

$$\frac{\sigma}{\sqrt{n}} = \frac{11.3}{\sqrt{81}} = \frac{11.3}{9} = 1.25$$

For a 90% confidence interval: $z_{\alpha/2} = z_{0.05} = 1.645$

$$CI = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 74.6 \pm 1.645 \times 1.2556 = 74.6 \pm 2.0654$$

$$\boxed{(72.5346, \quad 76.6654)}$$

4. Let $X_1, \ldots, X_n, X_{n+1}$ be a sample from a normal population having an unknown mean μ and variance 1. Let

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

be the average of the first n of them.

(a) What is the distribution of $X_{n+1} - \overline{X}_n$?

(b) If $\overline{X}_n = 4$, give an interval that, with 90% confidence, will contain the value of X_{n+1}.

Solution: $X_1, \ldots, X_n, X_{n+1}$ from a Normal population with unknown mean μ and variance 1. Let $\bar{X}_n = \sum_{i=1}^{n} X_i/n$ be the average of the first n observations.

(a) Distribution of $X_{n+1} - \bar{X}_n$

Since all X_i are i.i.d. $N(\mu, 1)$:

Mean:

$$E\big[X_{n+1} - \bar{X}_n\big] = \mu - \mu = 0$$

Variance (X_{n+1} is independent of $\bar{X}_n$, since X_{n+1} is not part of $\bar{X}_n$):

$$\text{Var}\big(X_{n+1} - \bar{X}_n\big) = \text{Var}(X_{n+1}) + \text{Var}(\bar{X}_n) = 1 + \frac{1}{n} = \frac{n+1}{n}$$

Therefore:

$$\boxed{X_{n+1} - \bar{X}_n \;\sim\; N\left(0, \; \frac{n+1}{n}\right)}$$

(b) 90% Prediction Interval for X_{n+1} given $\bar{X}_n = 4$

We standardize:

$$Z = \frac{X_{n+1} - \bar{X}_n}{\sqrt{(n+1)/n}} \;\sim\; N(0,1)$$

For a 90% confidence level: $z_{\alpha/2} = z_{0.05} = 1.645$

$$P\left(-1.645 \ \le\ \frac{X_{n+1} - \bar{X}_n}{\sqrt{(n+1)/n}} \ \le\ 1.645\right) = 0.90$$

Substituting $\bar{X}_n = 4$:

$$\boxed{X_{n+1} \ \in\ \left(4 - 1.645\sqrt{\frac{n+1}{n}},\quad 4 + 1.645\sqrt{\frac{n+1}{n}}\right)}$$

Note: As $n \to \infty$, $\sqrt{(n+1)/n} \to 1$, so the interval approaches $(4 - 1.645,\ 4 + 1.645) = (2.355,\ 5.645)$.

5. If $X_1, \ldots, X_n$ is a sample from a normal population whose mean μ is unknown but whose variance σ^2 is known, show that

$$\left(-\infty, \overline{X} + z_\alpha \frac{\sigma}{\sqrt{n}}\right)$$

is a $100(1-\alpha)\%$ lower confidence interval for μ.

Solution: $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, σ^2 known, μ unknown.

Claim: $\left(-\infty,\ \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right)$ is a $100(1-\alpha)\%$ lower confidence interval for μ.

Proof.

Since $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, we standardize:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \ \sim\ N(0,1)$$

By definition of z_α (upper α critical value):

$$P(Z \le z_\alpha) = 1 - \alpha$$

Since $N(0,1)$ is symmetric, $P(Z \ge -z_\alpha) = P(Z \le z_\alpha) = 1 - \alpha$.

Substituting the expression for Z:

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \ge -z_\alpha\right) = 1 - \alpha$$

Rearranging the inequality inside the probability:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \ge -z_\alpha \ \Leftrightarrow\ \bar{X} - \mu \ge -z_\alpha \cdot \frac{\sigma}{\sqrt{n}} \ \Leftrightarrow\ \mu \le \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

Therefore:

$$P\left(\mu \le \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

which is equivalently written as:

$$\boxed{P\left(\mu \ \in\ \left(-\infty,\ \bar{X} + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right)\right) = 1 - \alpha}$$

This confirms that $\left(-\infty,\ \bar{X} + z_\alpha\, \sigma/\sqrt{n}\right)$ is a valid $100(1-\alpha)\%$ lower confidence interval for μ. ■

6. A sample of 20 cigarettes is tested to determine nicotine content and the average value observed was 1.2 mg. Compute a 99% two-sided confidence interval for the mean nicotine content of a cigarette if it is known that the standard deviation of a cigarette's nicotine content is $\sigma = 0.2$ mg.

   Solution: $n = 20$, $\bar{x} = 1.2$ mg, $\sigma = 0.2$ mg, Confidence level = 99%

$$\frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{20}} = \frac{0.2}{4.4721} = 0.04472$$

   For a 99% two-sided confidence interval:

$$z_{\alpha/2} = z_{0.005} = 2.576$$

$$CI = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.2 \pm 2.576 \times 0.04472 = 1.2 \pm 0.1152$$

$$\boxed{(1.0848 \text{ mg}, \quad 1.3152 \text{ mg})}$$

   We are 99% confident that the true mean nicotine content of a cigarette lies between **1.0848 mg** and **1.3152 mg**.

7. In Problem 13, suppose that the population variance is not known in advance of the experiment. If the sample variance is 0.04, compute a 99% two-sided confidence interval for the mean nicotine content.

   Solution: From Problem 13: $n = 20$, $\bar{x} = 1.2$ mg, but now σ^2 is **unknown**. Sample variance $s^2 = 0.04$, so $s = \sqrt{0.04} = 0.2$ mg. Confidence level = 99%.

   Since σ^2 is **unknown**, we use the **t-distribution** with $\nu = n - 1 = 19$ degrees of freedom.

$$\frac{s}{\sqrt{n}} = \frac{0.2}{\sqrt{20}} = \frac{0.2}{4.4721} = 0.04472$$

   For a 99% two-sided confidence interval:

$$t_{\alpha/2, n-1} = t_{0.005, 19} = 2.861$$

$$CI = \bar{x} \pm t_{0.005, 19} \cdot \frac{s}{\sqrt{n}} = 1.2 \pm 2.861 \times 0.04472 = 1.2 \pm 0.1279$$

$$\boxed{(1.0721 \text{ mg}, \quad 1.3279 \text{ mg})}$$

   Comparison with Problem 13: When σ was known, the 99% CI was (1.0848, 1.3152) using $z = 2.576$. With unknown variance, using $t_{0.005,19} = 2.861$ gives a **wider interval**, reflecting the extra uncertainty introduced by estimating σ from the data.

8. The following data resulted from 24 independent measurements of the melting point of lead:

$$\begin{gathered} 330°\text{C},\ 322°\text{C},\ 345°\text{C}, \\ 328.6°\text{C},\ 331°\text{C},\ 342°\text{C}, \\ 342.4°\text{C},\ 340.4°\text{C},\ 329.7°\text{C}, \\ 334°\text{C},\ 326.5°\text{C},\ 325.8°\text{C}, \\ 337.5°\text{C},\ 327.3°\text{C},\ 322.6°\text{C}, \\ 341°\text{C},\ 340°\text{C},\ 333°\text{C}, \\ 343.3°\text{C},\ 331°\text{C},\ 341°\text{C}, \\ 329.5°\text{C},\ 332.3°\text{C},\ 340°\text{C}. \end{gathered}$$

Assuming that the measurements can be regarded as constituting a normal sample whose mean is the true melting point of lead, determine a 95% two-sided confidence interval for this value. Also determine a 99% two-sided confidence interval.

Solution: 24 independent measurements, σ^2 unknown $\Rightarrow$ use **t-distribution** with $\nu = n-1 = 23$ degrees of freedom.

$$\sum_{i=1}^{24} x_i = 7956.9 \qquad \Rightarrow \qquad \bar{x} = \frac{7956.9}{24} = 331.538\,^\circ\mathrm{C}$$

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{1257.365}{23} = 54.668 \qquad \Rightarrow \qquad s = \sqrt{54.668} = 7.394\,^\circ\mathrm{C}$$

$$\frac{s}{\sqrt{n}} = \frac{7.394}{\sqrt{24}} = \frac{7.394}{4.899} = 1.509$$

(a) 95% Two-Sided Confidence Interval

Critical value: $t_{0.025,\,23} = 2.069$

$$CI = \bar{x} \pm t_{0.025,\,23} \cdot \frac{s}{\sqrt{n}} = 331.538 \pm 2.069 \times 1.509 = 331.538 \pm 3.122$$

$$\boxed{(328.416\,^\circ\mathrm{C}, \quad 334.660\,^\circ\mathrm{C})}$$

(b) 99% Two-Sided Confidence Interval

Critical value: $t_{0.005,\,23} = 2.807$

$$CI = \bar{x} \pm t_{0.005,\,23} \cdot \frac{s}{\sqrt{n}} = 331.538 \pm 2.807 \times 1.509 = 331.538 \pm 4.236$$

$$\boxed{(327.302\,^\circ\mathrm{C}, \quad 335.774\,^\circ\mathrm{C})}$$

9. The following are scores on IQ tests of a random sample of 18 students at a large eastern university:

130, 122, 119, 142, 136, 127, 120, 152, 141,

132, 127, 118, 150, 141, 133, 137, 129, 142.

(a) Construct a 95% confidence interval estimate of the average IQ score of all students at the university.
(b) Construct a 95% lower confidence interval estimate.
(c) Construct a 95% upper confidence interval estimate.

Solution: $n = 18$, σ^2 unknown $\Rightarrow$ use **t-distribution** with $\nu = n-1 = 17$ degrees of freedom.

$$\sum_{i=1}^{18} x_i = 2398 \qquad \Rightarrow \qquad \bar{x} = \frac{2398}{18} = 133.222$$

$$s^2 = \frac{1773.106}{17} = 104.300 \qquad \Rightarrow \qquad s = \sqrt{104.300} = 10.213$$

$$\frac{s}{\sqrt{n}} = \frac{10.213}{\sqrt{18}} = \frac{10.213}{4.243} = 2.408$$

(a) 95% Two-Sided Confidence Interval

Critical value: $t_{0.025,\,17} = 2.110$

$$CI = \bar{x} \pm t_{0.025,\,17} \cdot \frac{s}{\sqrt{n}} = 133.222 \pm 2.110 \times 2.408 = 133.222 \pm 5.081$$

$$\boxed{(128.141, \quad 138.303)}$$

(b) 95% Lower Confidence Interval (One-Sided Lower Bound)

Critical value: $t_{0.05,\,17} = 1.740$

$$\bar{x} - t_{0.05,\,17} \cdot \frac{s}{\sqrt{n}} = 133.222 - 1.740 \times 2.408 = 133.222 - 4.190 = 129.032$$

$$\boxed{\mu \;\geq\; 129.032}$$

(c) 95% Upper Confidence Interval (One-Sided Upper Bound)

$$\bar{x} + t_{0.05,\,17} \cdot \frac{s}{\sqrt{n}} = 133.222 + 1.740 \times 2.408 = 133.222 + 4.190 = 137.412$$

$$\boxed{\mu \;\leq\; 137.412}$$

10. An important issue for a retailer is to decide when to reorder stock from a supplier. A common policy used to make the decision is of a type called (s, S). The retailer orders at the end of a period if the on-hand stock is less than s, and orders enough to bring the stock up to S. The appropriate values of s and S depend on different cost parameters, such as inventory holding costs and the profit per item sold, as well as the distribution of the demand during a period. Consequently, it is important for the retailer to collect data relating to the parameters of the demand distribution.

Suppose that the following data give the numbers of a certain type of item sold in each of 30 weeks:

$$14, 8, 12, 9, 5, 22, 15, 12, 16, 7, 10, 9, 15, 15, 12,$$

$$9, 11, 16, 8, 7, 15, 13, 9, 5, 18, 14, 10, 13, 7, 11$$

Assuming that the numbers sold each week are independent random variables from a common distribution, use the data to obtain a 95% confidence interval for the mean number sold in a week.

Solution: $n = 30$ weekly sales figures, σ^2 unknown $\Rightarrow$ use **t-distribution** with $\nu = n - 1 = 29$ degrees of freedom.

$$\sum_{i=1}^{30} x_i = 347$$

$$\bar{x} = \frac{347}{30} = 11.567$$

$$s^2 = \frac{\sum_{i=1}^{30}(x_i - \bar{x})^2}{n-1} = \frac{458.170}{29} = 15.799 \qquad \Rightarrow \qquad s = \sqrt{15.799} = 3.975$$

$$\frac{s}{\sqrt{n}} = \frac{3.975}{\sqrt{30}} = \frac{3.975}{5.477} = 0.7259$$

For a 95% two-sided confidence interval with $\nu = 29$ degrees of freedom:

$$t_{\alpha/2,\,n-1} = t_{0.025,\,29} = 2.045$$

$$CI = \bar{x} \,\pm\, t_{0.025,\,29} \cdot \frac{s}{\sqrt{n}} = 11.567 \pm 2.045 \times 0.7259 = 11.567 \pm 1.484$$

$$\boxed{(10.083, \quad 13.051)}$$

We are 95% confident that the true mean number of items sold per week lies between **10.083** and **13.051** units.

11. Find a 95% two-sided confidence interval for the variance of the diameter of a rivet based on the data given here.

6.68	6.66	6.62	6.72
6.76	6.67	6.70	6.72
6.78	6.66	6.76	6.72
6.76	6.70	6.76	6.76
6.74	6.74	6.81	6.66
6.64	6.79	6.72	6.82
6.81	6.77	6.60	6.72
6.74	6.70	6.64	6.78
6.70	6.70	6.75	6.79

Assume a normal population.

Solution: 36 measurements of rivet diameter from a normal population, σ^2 unknown $\Rightarrow$ use the **Chi-squared distribution**.

The pivot statistic for a CI on variance is:

$$\frac{(n-1)\,s^2}{\sigma^2} \;\sim\; \chi^2_{n-1}$$

$$\sum_{i=1}^{36} x_i = 242.73 \qquad \Rightarrow \qquad \bar{x} = \frac{242.73}{36} = 6.7425$$

$$s^2 = \frac{\sum_{i=1}^{36}(x_i - \bar{x})^2}{n-1} = \frac{0.123900}{35} = 0.003540 \qquad \Rightarrow \qquad s = \sqrt{0.003540} = 0.05950$$

For a 95% two-sided CI with $\nu = n - 1 = 35$ degrees of freedom:

$$\chi^2_{0.025,\,35} = 53.203 \qquad \text{and} \qquad \chi^2_{0.975,\,35} = 20.569$$

Starting from the pivot:

$$P\left(\chi^2_{0.975,\,35} \;\le\; \frac{(n-1)\,s^2}{\sigma^2} \;\le\; \chi^2_{0.025,\,35}\right) = 0.95$$

Rearranging for σ^2:

$$P\left(\frac{(n-1)\,s^2}{\chi^2_{0.025,\,35}} \;\le\; \sigma^2 \;\le\; \frac{(n-1)\,s^2}{\chi^2_{0.975,\,35}}\right) = 0.95$$

Substituting values:

$$\left(\frac{35 \times 0.003540}{53.203},\quad \frac{35 \times 0.003540}{20.569}\right) = \left(\frac{0.12390}{53.203},\quad \frac{0.12390}{20.569}\right)$$

$$\boxed{\sigma^2 \;\in\; (0.002329, \quad 0.006024)}$$

Taking square roots of both bounds:

$$\boxed{\sigma \;\in\; \left(\sqrt{0.002329}, \quad \sqrt{0.006024}\right) = (0.04826, \quad 0.07762)}$$

12. A civil engineer wishes to measure the compressive strength of two different types of concrete. A random sample of 10 specimens of the first type yielded the following data (in psi)

Type 1: 3,250, 3,268, 4,302, 3,184, 3,266, 3,297, 3,332, 3,502, 3,064, 3,116

whereas a sample of 10 specimens of the second yielded the data

Type 2: 3,094, 3,106, 3,004, 3,066, 2,984, 3,124, 3,316, 3,212, 3,380, 3,018

If we assume that the samples are normal with a common variance, determine

(a) a 95% two-sided confidence interval for $\mu_1 - \mu_2$, the difference in means;

(b) a 95% one-sided upper confidence interval for $\mu_1 - \mu_2$;

(c) a 95% one-sided lower confidence interval for $\mu_1 - \mu_2$.

Solutions: Two independent normal samples with a **common (pooled) variance**; $n_1 = n_2 = 10$. Use the **pooled two-sample t-statistic** with $\nu = n_1 + n_2 - 2 = 18$ degrees of freedom.

$$\sum x_{1i} = 33{,}581 \qquad \Rightarrow \qquad \bar{x}_1 = \frac{33{,}581}{10} = 3358.1$$

$$\sum x_{2i} = 31{,}304 \qquad \Rightarrow \qquad \bar{x}_2 = \frac{31{,}304}{10} = 3130.4$$

$$\bar{x}_1 - \bar{x}_2 = 3358.1 - 3130.4 = 227.7$$

$$s_1^2 = \frac{1{,}119{,}772.90}{9} = 124{,}419.21$$

$$s_2^2 = \frac{159{,}558.40}{9} = 17{,}728.71$$

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2} = \frac{9 \times 124{,}419.21 \ + \ 9 \times 17{,}728.71}{18} = \frac{1{,}279{,}331.30}{18} = 71{,}073.96$$

$$s_p = \sqrt{71{,}073.96} = 266.60$$

$$SE = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 266.60 \sqrt{\frac{1}{10} + \frac{1}{10}} = 266.60\sqrt{0.2} = 266.60 \times 0.4472 = 119.21$$

$$t_{0.025,\,18} = 2.101 \quad \text{(two-sided 95\%)} \qquad t_{0.05,\,18} = 1.734 \quad \text{(one-sided 95\%)}$$

(a) 95% Two-Sided CI for $\mu_1 - \mu_2$

$$CI = (\bar{x}_1 - \bar{x}_2) \ \pm \ t_{0.025,\,18} \cdot SE = 227.7 \pm 2.101 \times 119.21 = 227.7 \pm 250.46$$

$$\boxed{\mu_1 - \mu_2 \ \in \ (-22.76, \quad 478.16)}$$

Since 0 lies inside this interval, we **cannot conclude** at 95% confidence that the two concrete types differ in mean compressive strength.

(b) 95% One-Sided Upper CI for $\mu_1 - \mu_2$

Upper bound: we assert $\mu_1 - \mu_2$ is **at most**:

$$(\bar{x}_1 - \bar{x}_2) + t_{0.05,\,18} \cdot SE = 227.7 + 1.734 \times 119.21 = 227.7 + 206.71 = 434.41$$

$$\boxed{\mu_1 - \mu_2 \ \leq \ 434.41}$$

(c) 95% One-Sided Lower CI for $\mu_1 - \mu_2$

Lower bound: we assert $\mu_1 - \mu_2$ is **at least**:

$$(\bar{x}_1 - \bar{x}_2) - t_{0.05,\,18} \cdot SE = 227.7 - 1.734 \times 119.21 = 227.7 - 206.71 = 20.99$$

$$\boxed{\mu_1 - \mu_2 \ \geq \ 20.99}$$

13. Independent random samples are taken from the output of two machines on a production line. The weight of each item is of interest. From the first machine, a sample of size 36 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 64 is taken, with a sample mean weight of 130 grams and a sample variance of 5. It is assumed that the weights of items from the first machine are normally distributed with mean μ_1 and variance σ^2 and that the weights of items from the second machine are normally distributed with mean μ_2 and variance σ^2 (that is, the variances are assumed to be equal). Find a 99% confidence interval for $\mu_1 - \mu_2$, the difference in population means.

Solution:
For Type I ($n_1 = 10$):
Sample Mean: $\bar{x}_1 = 532.1$, Sample Variance: $s_1^2 = \frac{\sum(x_i-\bar{x}_1)^2}{n_1-1} \approx 2932.77$
For Type II ($n_2 = 10$):
Sample Mean: $\bar{x}_2 = 548.6$, Sample Variance: $s_2^2 = \frac{\sum(x_i-\bar{x}_2)^2}{n_2-1} = 1191.6$
Under the assumption of equal variances:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$
$$s_p^2 = \frac{9(2932.77) + 9(1191.6)}{18}$$
$$s_p^2 = 2062.18$$

For a 99% confidence interval with $df = 10 + 10 - 2 = 18$ degrees of freedom:

$$t^* = t_{0.005,18} \approx 2.878$$

The confidence interval is given by:

$$CI = (\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Substituting the values:

$$CI = (532.1 - 548.6) \pm 2.878 \sqrt{2062.18 \left(\frac{1}{10} + \frac{1}{10}\right)}$$
$$CI = -16.5 \pm 2.878\sqrt{412.436}$$
$$CI = -16.5 \pm 2.878(20.31)$$
$$CI = -16.5 \pm 58.46$$

Final Interval: $[-74.96, 41.96]$