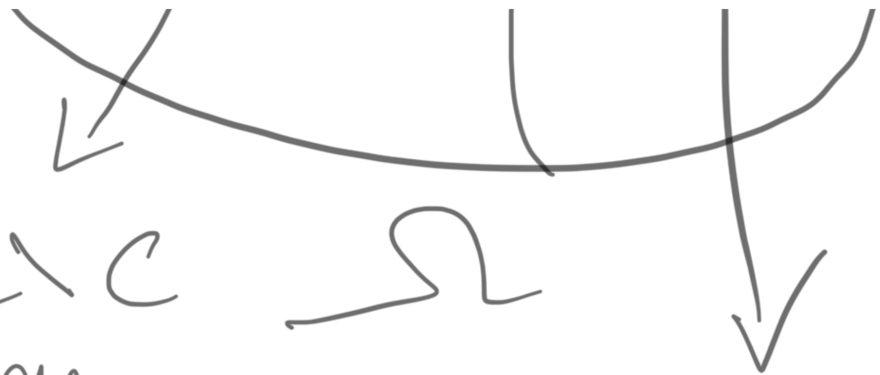
Apr 17

H_0	True	False
Accept	✓	Type-II
Reject	Type-I	✓

Critical Region



$\Omega \setminus C$ Ω

region of acceptance

If $X \in C$ we reject H_0

Neyman Pearson Lemma

$X_1, \ldots, X_n$ is a random sample from a pop. with PMF/PDF $f_0(\cdot)$.

want to test

We

$H_0 : \theta = \theta_0$

$H_1 : \theta = \theta_1$

The most powerful test

$$\frac{L(\underset{\sim}{X}, \theta_0)}{L(\underset{\sim}{X}, \theta_1)} \le k \text{ then reject } H_0$$

$> k$ do not reject H_0

X_n

Eg. Let $X_1, \ldots, $

be random sample

from Bernoulli(θ).

we want test

$H_0 : \theta = 0.5$

against

$H_1 : \theta = 0.8$

Find the Most power-ful test (MP-test)

or Neymen - Pearson test. at level α.

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the likelihood for θ

is given by

$$L(\underset{\sim}{x}, \theta) = \prod_{i=1}^{n} \theta^{x_i} (1-\theta)^{1-x_i}$$

$$= \theta^{\sum x_i} (1-\theta)^{n-\sum x_i}$$

So, the MP test using NP lemma (Neyman Pearson lemma), we reject H_0 if

$$\frac{L(\underset{\sim}{x}, \theta_0)}{L(\underset{\sim}{x}, \theta_1)} \le k$$

$$\Rightarrow \frac{\theta_0^{\sum x_i} (1-\theta_0)^{n-\sum x_i}}{\theta_1^{\sum x_i} (1-\theta_1)^{n-\sum x_i}} \le k$$

Let's simplify this

$$\Rightarrow \left(\frac{\theta_0}{\theta_1}\right)^{\sum X_i} \left(\frac{1-\theta_0}{1-\theta_1}\right)^{n-\sum X_i} \leq k$$

$$\Rightarrow \left(\frac{\theta_0/1-\theta_0}{\theta_1/1-\theta_1}\right)^{\sum X_i} \underbrace{\left(\frac{1-\theta_0}{1-\theta_1}\right)^{n}}_{} \leq k$$

more

$\Rightarrow$ Then take log, simplify

$$\underline{\sum X_i} \leq k_1$$

k_1 is function of $k, \theta_0, \theta, \& n$

So, the rejection region is

$$C = \left\{ \underline{x} : \sum_{i=1}^{n} x_i \le k_1 \right\}$$

We know that for parameter θ,

$$\sum_{i=1}^{n} X_i \sim \text{Binomial}(n, \theta)$$

Using the definition of significance

$$\alpha = \mathbb{P}(\underline{X} \in C \mid H_0)$$

$$= \mathbb{P}\left(\sum_{i=1}^{n} X_i \le k_1 \mid \theta = \theta_0 \right)$$

$$= \mathbb{P}\left(\underbrace{\sum_{i=1} X_i}\right.$$

$$= \mathbb{P}\left(Y \le x_1 \;\middle|\; Y \sim \text{Binomial}(n, \theta_0)\right)$$

The p-value Approach

p-value is defined as

p-val

$$= \mathbb{P}\left(\cancel{\text{rejecting } H_0} \;\middle|\; H_0 \text{ is true}\right)$$

getting sample at least as extreme as $\underline{x}$

Rejection rule:

Reject H_0 for too small

p-val

or

for a given significance level α

Reject H_0, if p-val $< \alpha$