Apr 18

$H_0 : \mu = \mu_0$

$H_1 : \mu = \mu_1 > \mu_0$

$\beta = \mathbb{P}(H_0 \text{ is accepted} \mid H_1 \text{ is true})$

$= \mathbb{P}\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \leq Z_\alpha \;\middle|\; \mu = \mu_1 \right)$

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \leq Z_\alpha$$

$$\Rightarrow \frac{\bar{X} - \mu_1 + \mu_1 - \mu_0}{\sigma/\sqrt{n}} \leq Z_\alpha$$

$$\Rightarrow \frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} + \frac{\mu_1 - \mu_0}{\sigma/\sqrt{n}} \leq Z_\alpha$$

$$\Rightarrow \frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} \leq Z_\alpha + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}}$$

$$\frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} \sim N(0,1)$$

" Z_{test} follows $N(0, \ldots)$ if $\mu = \mu_1$

$$\Rightarrow \beta = \mathbb{P}\left(Z_{test} \le z_\alpha + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} \right)$$

$$= \Phi\left(z_\alpha + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} \right)$$

Power

$$\text{Power} = 1 - \beta = 1 - \beta(\mu_1)$$

$$= 1 - \Phi\left(z_\alpha + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} \right)$$

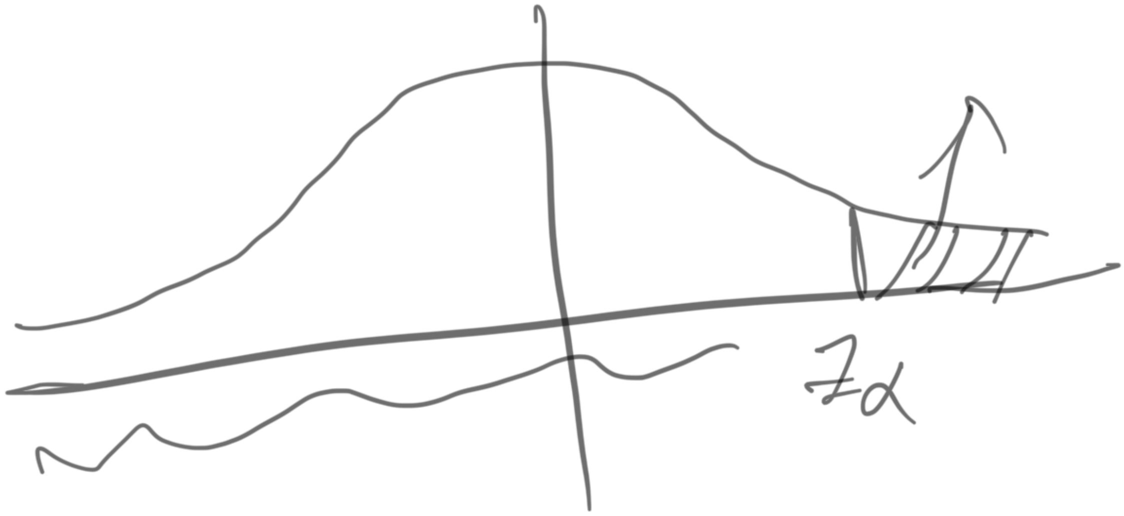
Power function as function of $\mu_1 = \theta$,

$P(\theta)$

$$\pi_\theta = 1 - \beta(\theta)$$
$$= 1 - \Phi\left(z_\alpha + \frac{\mu_0 - \theta}{\sigma/\sqrt{n}}\right)$$

when $\theta = \mu_0$

$$\pi_{\mu_0} = 1 - \Phi(z_\alpha)$$
$$= 1 - (1-\alpha) = \alpha$$



When θ is increasing to ∞.

$$\lim_{\theta\to\infty} \pi_\theta = 1 - \lim_{\theta\to\infty} \Phi\left(z_\alpha + \underbrace{\frac{\mu_0-\theta}{\sigma/\sqrt{n}}}\right)$$

$= 1 - 0$

$= 1$

For larger value of θ, test has

higher power

Lemma: If $\theta_1 > \theta_2$

then $\pi_{\theta_1} > \pi_{\theta_2}$

proof: Use monotonicity of CDF $\Phi(\cdot)$.

Testing variance of normal

Testing ... distribution

Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population.

We want to test

$$H_0 : \sigma^2 = \sigma_0^2$$

vs

$$H_1 : \sigma^2 \neq \sigma_0^2$$

Case I. μ is known.

Define $Y_i = \dfrac{X_i - \mu}{\sigma} \sim N(0,1)$

Then ... iid $N(0,1)$

Then, $Y_1, \dots, Y_n$ are i.i.d ...

So, $\sum_{i=1}^{n} Y_i^2 \sim \chi^2_n$

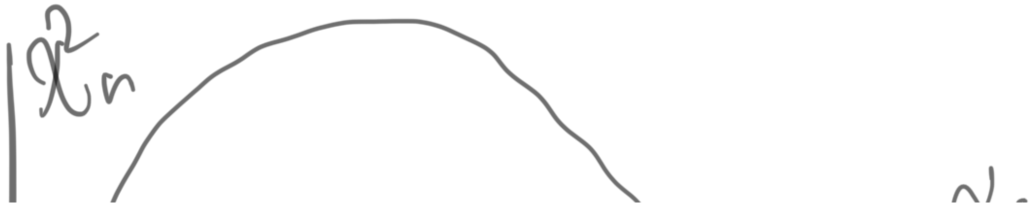
$$\Rightarrow \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2_n$$

$$\Rightarrow \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \mu)^2 \sim \chi^2_n$$

Under H_0,

$$T = \frac{1}{\sigma_0^2} \sum_{i=1}^{n} (X_i - \underline{\mu})^2 \sim \chi^2_n$$

pdf χ^2_n



Rejection region

$$C = \{ \underset{\sim}{x} : T < q_1 \text{ or } T > q_2 \}$$

When $H_1 : \sigma^2 = \sigma_1^2 > \sigma_0^2$