Jan 23

Continuous RV:

Def: Let X be a RV with CDF F. If F is a continuous function then X is said to be a continuous RV.

Types:

(i) Absolutely cont. RV

(ii) Singular cont. RV

Out of scope

Remark: In this course

Continuous RV
mean
Absolutely cont. RV

Probability density function (PDF)

For any continuous RV X on $(\mathbb{R}, \mathbb{B}(\mathbb{R}))$ with CDF F, there exists a $f : \mathbb{R} \to \mathbb{R}$ such that for any $x \in \mathbb{R}$,

$$F(x) = \int_{-\infty}^{x} f(x)\, dx.$$

The function f is known as PDF of RV X.

Eg: $\Omega = (0,1)$

$\mathcal{F} = \mathbb{B}((0,1))$

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Observe that

$$F(x) = \int^{x} f(x)\, dx$$

$$F(x) = \int_{-\infty} f(x)$$

where

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0, & \text{otherwise} \end{cases}$$

Comparison table

	RV	
Characterisation	Discrete	Continuous
CDF	✓	✓
PMF	✓	✗
PDF	✗	✓
MGF	✓	✓

MGF	✓	✓

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eg. of Discrete RV

① **Bernoulli RV:**

A random variable X with support set $\{0, 1\}$ and PMF

$$\mathbb{P}(X = x) = \begin{cases} 1-p & \text{if } x = 0 \\ p & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

is known as Bernoulli RV with parameter p.

Here $p \in (0, 1)$.

Associated CDF

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

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(11) <u>Binomial RV</u> :

A RV X is said to follow Binomial distribution if it has following PMF

$$\mathbb{P}(X=x) = \begin{cases} \binom{n}{x} p^{x} (1-p)^{n-x} & \text{if } x \in \{0,1,\ldots,n\} \\ 0 & \text{otherwise} \end{cases}$$

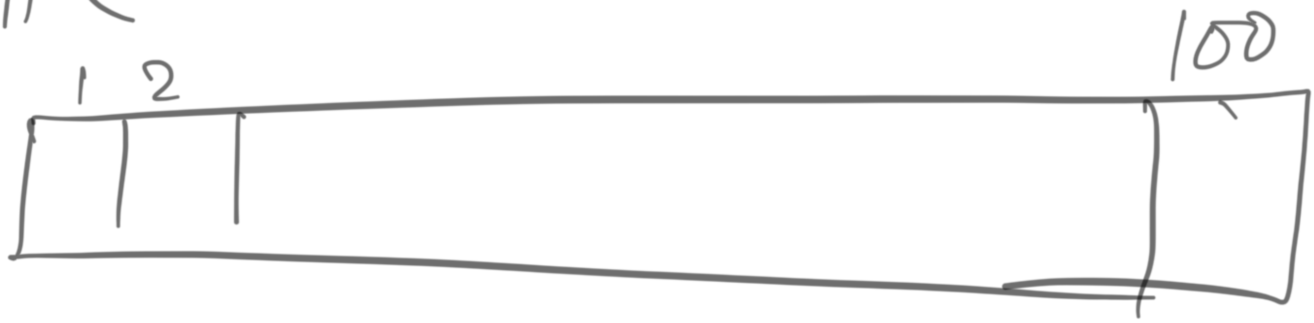
Here $n \in \mathbb{N}$, $p \in (0,1)$, and

$(\underline{n, p})$ are parameters of the distribution

Eg. In 100 tosses of the same coin, independently,

$X = \#$ heads

$P(X=2)$



$$= \binom{100}{2} p^2 (1-p)^{98}$$

$P(X = r)$

$$\left(100\right) p^r (1-p)^{n-r}$$

$= \binom{100}{r} \ldots$

$r \in \{0, 1, 2, \ldots, 100\}$

Expectation:

① Discrete RV

Let X be a discrete RV with PMF p.

Let h be a function of X.

The expectation of $h(X)$ is denoted by $\mathbb{E}(h(X))$ and defined as

$$\mathbb{E}(h(X)) = \sum_{x \in \text{support}} h(x) \cdot p(x)$$

Eg: let X is Bernoulli RV with parameter p.

$$\mathbb{E}(X) = \sum_{x \in \{0,1\}} x \cdot p(x)$$

$$= 0 \cdot (1-p) + 1 \cdot p$$

$$= p$$

$$\mathbb{E}(X^2) = \sum_{x \in \{0,1\}} x^2 \cdot p(x)$$

$$= 0^2 \cdot (1-p) + 1^2 \cdot p$$

$$= p$$

(11) Expectation of a function of cont. RV.

Let X be a cont. RV with PDF f. Let h be a function of X. The expected value of $h(X)$ is denoted by $\mathbb{E}(h(X))$ and defined as

$$\mathbb{E}(h(X)) = \int_{-\infty}^{\infty} h(x) \cdot f(x)\, dx$$

Eg: let X be a cont RV with PDF

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0 & \text{otherwise} \end{cases}$$

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\, dx$$

$$= \int_0^1 h(x) f(x)\, dx$$

$$= \int_0^1 h(x)\, dx$$

Remark:

① $p(x)$ is a valid PMF

if ⓐ $p(x) \geq 0$

(b) $\sum_{x} p(x) = 1$

(ii) $f(x)$ is a valid PDF

if (a) $f(x) \geq 0$

(b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Moments of a RV, $r \in \mathbb{N}$

Def: r^{th} moment of a RV X is defined as

$\mathbb{E}(X^r)$.

Eg X follows Bernoulli with parameter p:

first moment

$\mathbb{E}(X) = p$

second moment

$\mathbb{E}(X^2) = p$

Mean of a Random variable

Def: Mean of a RV X is $\mathbb{E}(X)$.

Eg: $\dfrac{100+100+100}{3} = 100$

$\dfrac{90+100+110}{3} = 100$

Variance of a RV

Def: Variance of a RV X is denoted by $Var(X)$ and defined by

$$Var(X) = E\left[(X - E(X))^2\right]$$